

Chapter 3

Linear and Quadratic Functions

Section 1

Linear Functions, Their Properties, and Linear Models

When an equation of a line is written in slope-intercept form with function notation we call it a **linear function**

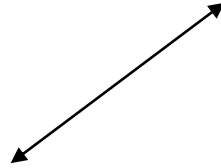
$$f(x) = mx + b \quad (m = \text{slope}, b = \text{y-intercept})$$

Average Rate of change of a linear function is constant and its known as slope:

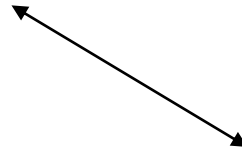
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \frac{y_1 - y_2}{x_1 - x_2}$$

Determining if increasing, decreasing, or constant is dependent on slope!

Increasing = positive slope



Decreasing = negative slope



Constant = zero slope

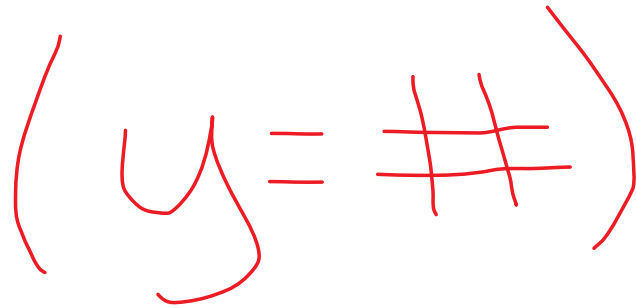


Modeling with a Linear Function

If the average rate of change is a constant m , a linear function $f(x)$ can be used to model the relation between the two variables as follows:

$$f(x) = mx + b$$

Where b is the value of f at 0, that is $f(0) = b$



A handwritten red equation $y = \#$ enclosed in large red parentheses. The '#' symbol is drawn with multiple horizontal and vertical strokes, resembling a hash or a placeholder for a value.

Example 1: (# 21 pg. 133)

Determine whether the given function is linear or nonlinear. If it is linear, determine the equation of the line.

X	Y
-2	4
-1	1
0	-2
1	-5
2	-8

$X \rightarrow +1, Y \rightarrow -3$

\rightarrow IS linear

$$m = \frac{4-1}{-2-(-1)} \rightarrow \frac{3}{-1} = -3$$

$$y = -3x - 2$$

Example 2: (#29 pg. 133)

$$f(x) = 4x - 1$$

$$g(x) = -2x + 5$$

a) Solve $f(x) = 0$

$$4x - 1 = 0$$

$$4x = 1$$

$$x = 1/4$$

b) Solve $f(x) > 0$

$$4x - 1 > 0$$

$$4x > 1$$

$$x > 1/4$$

c) Solve $f(x) = g(x)$

$$4x - 1 = -2x + 5$$

$$6x - 1 = 5$$

$$6x = 6$$

$$x = 1$$

d) Solve $f(x) \leq g(x)$

$$4x - 1 \leq -2x + 5$$

$$6x - 1 \leq 5$$

$$6x \leq 6$$

$$x \leq 1$$

Example 2 continued: (#29 pg. 133)

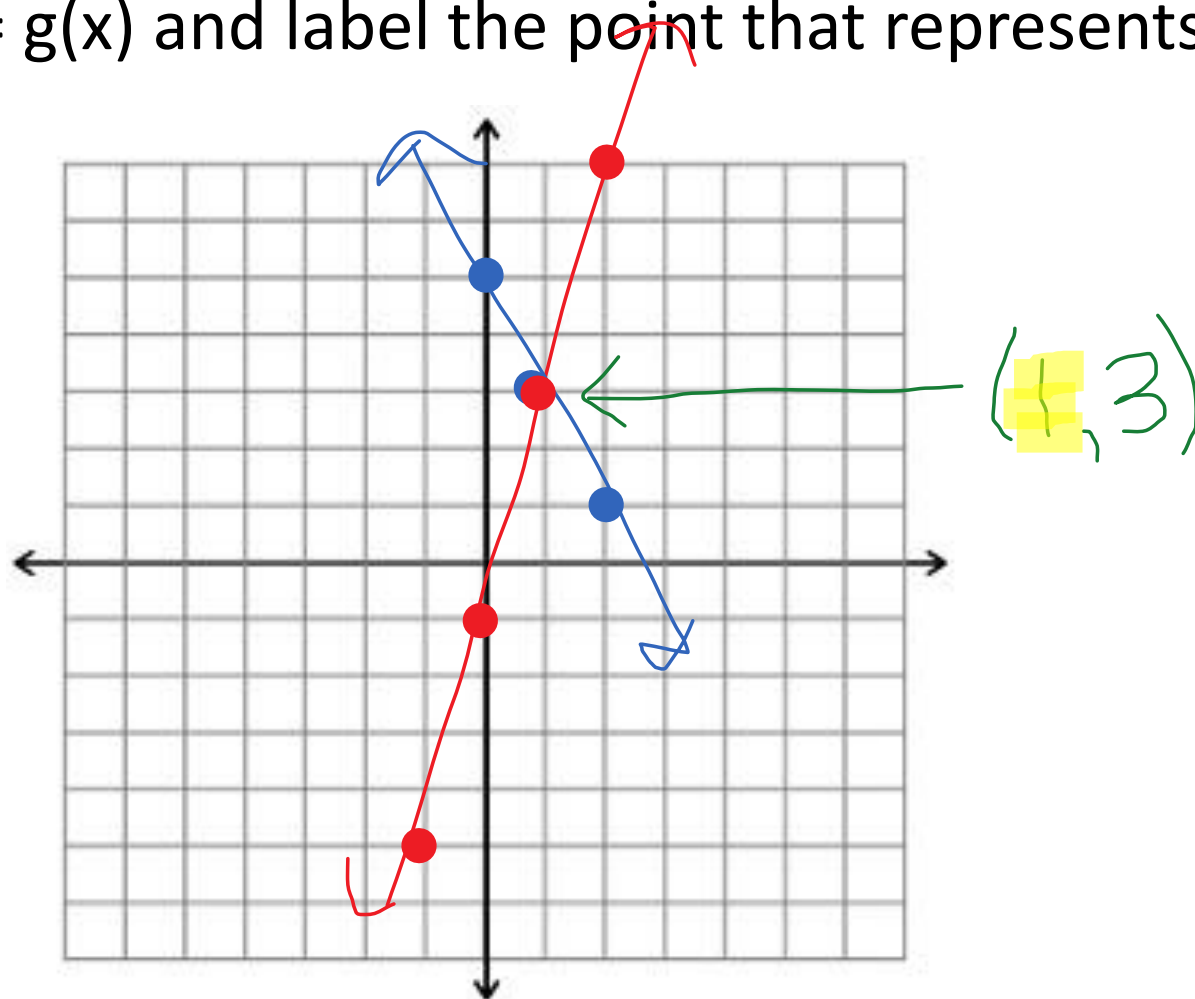
$$f(x) = 4x - 1$$

$$g(x) = -2x + 5$$

$$\hookrightarrow m = 4, b = -1$$

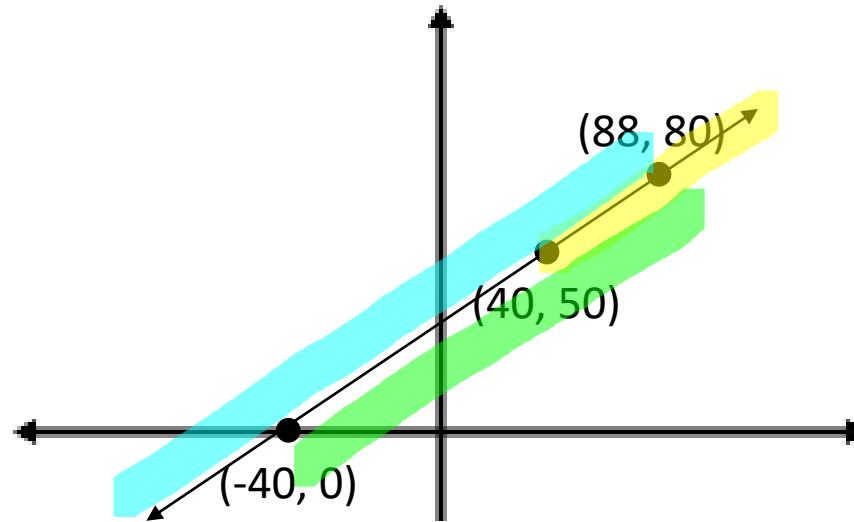
$$\hookrightarrow m = -2, b = 5$$

e) Graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution to the equation $f(x) = g(x)$.



Example 3: (#31 pg. 133)

Use the following figure:



a) Solve $f(x) = 50$ $x = 40$

b) Solve $f(x) = 80$ $x = 88$

c) Solve $f(x) = 0$ $x = -40$

d) Solve $f(x) > 50$ $x > 40$

e) Solve $f(x) \leq 80$ $x \leq 88$

f) Solve $0 < f(x) < 80$ $-40 < x < 88$

Example 4: (#37 pg. 134)

Car Rentals

$$C(x) = 0.25x + 35; x = \text{miles}, C = \text{cost}$$

What is the cost if you drive 40 miles?

$$C = .25(40) + 35$$

$$10 + 35$$

$$C = 45 \Rightarrow \$45$$

If the cost of renting the truck is \$80, how many miles did you drive?

$$80 = .25x + 35$$

$$45 = .25x$$

$$180 = x \Rightarrow 180 \text{ miles}$$

Example 4 continued: (#37 pg. 134)

Car Rentals

$$C(x) = 0.25x + 35; x = \text{miles}, C = \text{cost}$$

Suppose you want the cost to be no more than \$100, what is the maximum number of miles that you can drive?

$$100 \geq .25x + 35$$

$$65 \geq .25x$$

$$260 \geq x \Rightarrow \text{@ most 260 miles}$$

What is the implied domain of C?

$$\{x > 0\}$$

EXIT SLIP